DOUBLE RATIO ESTIMATE IN FOREST SURVEYS

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INTRODUCTION

IN forest sample surveys, estimation of the ratio of total number of trees of exploitable diameter class (*i.e.*, all trees above the minimum exploitable diameter) to the next lower diameter class, and of the total or mean number of trees of a particular diameter class are important problems. The former enables us to prescribe the annual yields by using the safeguarding formula for selection felling series given by Smythies (1933). Each felling series consists of a group of compartments of varying sizes. The primary sampling unit is a compartment or a section of it bounded by topographical features like ridges, nalas, etc. Diameter classes usually used are of size 4″, the classes being 8″–12″, 12″–16″, 16″–20″, 20″–24″ and so on. Occasionally 2″ diameter classes are also used.

The minimum exploitable diameter is fixed, since trees below this diameter are not saleable, while the area is so unstable that trees should be felled as soon as they are readily saleable. The trees over the minimum exploitable diameter are called class I trees and trees belonging to the diameter class just below the minimum exploitable diameter are called class II trees. Nair (1956) has illustrated the estimation of the ratio of class I to class II trees and its standard error and computed the safe probability limit of yield percentage by using Smythies formula for two working plan surveys of "Haldwani Sal Forests" of Uttar Pradesh and "Sal Forests of Porahat and Saranda Divisions", Bihar. In this paper, the use of a double ratio estimate to estimate the ratio of class I to class II trees, when previous complete enumeration data are available, is illustrated. Keyfitz (refer Yates, 1949) used double ratio estimate to estimate the total labour force, salaries, material used, etc., in the case in which there is an initial complete census of production, and of labour force, etc., and subsequently a further complete census of production, but a sample only for labour force. If the ratio of the ratios is less subject to variation than either ratio separately, double ratio estimate is more precise.

Estimation of the total or mean number of trees of lower diameter class is important, since it enables us to tell how many of the trees will grow up to the next higher diameter class and subsequently to the exploitable size during the felling cycle. Nair (1956) used regression method of estimation to estimate the mean number of trees of different diameter classes for "Kalagarah Working Plan", Uttar Pradesh, by utilising the supplementary information of the previous complete enumeration data. Since trees of lower diameter are normally more numerous than trees of higher diameter, and since a more precise estimate for the number of trees of higher diameter class (both class I and class II) is required, double sampling may be used, *i.e.*, a large sample of compartments is taken at random and number of trees of higher diameter class only are counted, and a sub-sample relatively small, is taken at random from the preliminary large sample and number of trees of lower diameter class are counted. The estimate and the variance for the mean number of trees of lower diameter class utilising the supplementary information of the number of trees of higher diameter, or the previous complete enumeration data, or both, are given. Also, utilising the supplementary information of trees of higher diameter of the present enumeration and of the areas of the compartments or topographical sections within each felling series, the mean and the variance of mean number of trees per acre of a particular lower diameter class are given. Also, efficiency of stratification for the double ratio estimate of the ratio is considered.

The formulæ given in this paper are valid for large size samples and should be used only for fairly large samples. But, there is need to develop suitable formulæ for medium size samples, since usually a felling series is composed of fifty to sixty compartments, and a sampling intensity of not more than 33 per cent. is chosen. The number of sampling units in a felling series can be increased by further dividing the compartments into sub-compartments of smaller size, but the location of these small units in the forest may become difficult and increase the cost of the survey. The author is investigating the bias of the double ratio estimate for small samples, by finding the second approximation to the variance, etc.

2. Estimation of the Ratio and the Annual Yield

Let, y_{1i} denote the number of trees of class I of the present enumeration in the ith compartment.

y_{2i} „ „ class II „ „

x_{1i} number of trees of class I of the previous complete enumeration in the i-th compartment.

x_{2i} ,, ,, class II ,, ,,

so that $X_1 = \sum_1^N x_{1i}$ and $X_2 = \sum_1^N x_{2i}$, the total number of trees of class I and class II of previous enumeration are known.

If a simple random sample of n compartments or topographical sections is drawn in the present enumeration, double ratio estimate of

$$R_y = \frac{Y_1}{Y_2} = \frac{\sum_1^N y_{1i}}{\sum_1^N y_{2i}}$$

is given by

$$R'_{yn} = \frac{\sum_1^n y_{1i} \Big/ \sum_1^n y_{2i}}{\sum_1^n x_{1i} \Big/ \sum_1^n x_{2i}} \cdot \frac{X_1}{X_2} = \frac{R_{yn}}{R_{xn}} \cdot R_x$$

where

$$R_{yn} = \frac{\text{Total number of trees of class I in the sample}}{\text{Total number of trees of class II in the same sample}}$$

of the present enumeration, and

$$R_{xn} = \frac{\text{Total number of trees of class I in the same sample}}{\text{Total number of trees of class II in the same sample}}$$

of the previous complete enumeration.

If there was no previous complete enumeration, estimate of R_y is R_{yn}.

Let,

$$R_n = \frac{R_{yn}}{R_{xn}} \quad \text{and} \quad R = \frac{R_y}{R_x}$$

$$\therefore \quad R'_{yn} - R_y = \frac{\bar{y}_1/\bar{y}_2}{\bar{x}_1/\bar{x}_2} \cdot \frac{\bar{X}_1}{\bar{X}_2} - \frac{\bar{Y}_1}{\bar{Y}_2}$$

$$= \frac{(\bar{X}_1/\bar{X}_2)}{(\bar{x}_1/\bar{x}_2)} \left\{ \frac{\bar{y}_1}{\bar{y}_2} - R\frac{\bar{x}_1}{\bar{x}_2} \right\}.$$

In what follows, we assume the sample size to be sufficiently large so that terms of order $O(1/n^{\kappa})$ where $\kappa > 1$ can be ignored.

$$\therefore \quad R'_{yn} - R_y \approx \left(\frac{\bar{y}_1}{\bar{y}_2} - R\frac{\bar{x}_1}{\bar{x}_2}\right) = R_{yn} - R \cdot R_{xn}$$

$$V(R'_{yn}) \approx V(R_{yn}) - 2R \text{ cov. } (R_{yn}R_{xn}) + R^2 V(R_{xn})$$

where

$$V(R_{yn}) = V\left(\frac{\bar{y}_1}{\bar{y}_2}\right) \approx \frac{1}{\bar{Y}_2^{\,2}}\left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{N-1}\sum_1^N (y_1 - R_2 y_2)^2$$

a similar expression for $V(R_{xn})$

and

$$\text{cov. } (R_{yn}R_{xn}) = \text{cov. } \left(\frac{\bar{y}_1}{\bar{y}_2}, \frac{\bar{x}_1}{\bar{x}_2}\right)$$

$$\approx \frac{1}{\bar{Y}_2\bar{X}_2}\left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{N-1}\sum_1^N (y_1 - R_y y_2) \times (x_1 - R_x x_2).$$

A consistent estimate of $V(R'_{yn})$ is

$$\hat{V}(R'_{yn}) \approx \left(\frac{N-n}{N}\right) \cdot \frac{n}{n-1}\left[\frac{\sum_1^n (y_1 - R_{yn}y_2)^2}{(\sum_1^n y_2)^2} - 2R_n \frac{\sum_1^n (y_1 - R_{yn}y_2)(x_1 - R_{xn}x_2)}{\left(\sum_1^n y_2\right)\left(\sum_1^n x_2\right)} + R_n^{\,2}\frac{\sum_1^n (x_1 - R_{xn}x_2)^2}{\left(\sum_1^n x_2\right)^2}\right] \quad (1)$$

A consistent estimate of $V(R_{yn})$ is

$$\hat{V}(R_{yn}) \approx \frac{N-n}{N} \cdot \frac{n}{n-1}\left[\frac{\sum_1^n (y_1 - R_{yn}y_2)^2}{\left(\sum_1^n y_2\right)^2}\right] \quad (2)$$

Estimate of the efficiency of the double ratio estimate of the ratio is given by

$$\frac{\hat{V}(R_{yn})}{\hat{V}(R'_{yn})}$$

% Annual yield 'y' is given by Smythies (1933) as

$$y = \frac{X}{I + \frac{X}{2}} \cdot 100$$

where

$$X = \frac{f}{t}(II - Z\% \text{ of } II)$$

X = actual number of trees of class II which survive and pass into class I during the felling cycle

f = felling cycle

t = period that class II trees take to grow to the exploitable size

Z = mortality per cent., *i.e.*, percentage of class II trees that disappear in 't' years

II = number of class II trees

I = number of class I trees.

Substituting for X in 'y' we get

$$y = \frac{200}{\left\{\left(\frac{200}{100 - Z}\right)\frac{t}{f} \cdot \frac{I}{II} + 1\right\}} \qquad (3)$$

so that if the estimate of the ratio of total number of class I to II trees is known, the annual yields can be prescribed for the values of t, f and Z assumed in the Working Plan. To compute the safe limit for the annual yield, we take the upper confidence limit for the ratio I/II calculated from R'_{yn} and its standard error.

Illustration.—The only data available with a previous complete enumeration are the enumeration data of Kalagarah Working Plan, U.P. Whole compartments were used as sampling units. In 1933–34, all the compartments were enumerated, and in 1952–53, a percentage of roughly 28% of the compartments were enumerated for 'Sal'. 20″ has been taken as the minimum exploitable diameter for the trees, so that I class trees are those having 20″-over diameter and II class trees are those having 16″–20″ diameter. The values of t, f and Z assumed in the Working Plan are 35 years, 15 years and 33% respectively.

There are four felling series, namely, Dhara Range, S.P. Dun Range, Mandal-Aduala Stratum and Mandal Stratum and the random samples taken in the present enumeration are of medium size.

Dhara Range ..	.. $N = 67, n = 12$
S.P. Dun Range ..	.. $N = 50, n = 18$
Mandal-Aduala Stratum	.. $N = 54, n = 12$
Mandal Stratum ..	.. $N = 55, n = 11$

Now, the question is whether the formulæ based on large samples can be applied to these felling series, to illustrate the efficiency of double ratio estimate of the ratio. Only for S.P. Dun Range, formulæ (1) and (2) seems to be satisfactory, since coefficient of variation of $\bar{y}_2$ and $\bar{x}_2$ are both $\cdot 07$ and coefficient of covariance of $\bar{y}_2$ and $\bar{x}_2$ is $\cdot 05$. Also, the first and second approximation to the variance of R_{yn} and R_{xn} are found out using the formulæ given in Sukhatme (1954).

$$\hat{V}_1 (R_{yn}) = \cdot 0010026 \qquad \hat{V}_1 (R_{xn}) = \cdot 001629$$

$$\hat{V}_2 (R_{yn}) = \cdot 0010368 \qquad \hat{V}_2 (R_{xn}) = \cdot 001664$$

where V_1 and V_2 are variances under first and second approximations respectively. So, formulæ (1) and (2) may be considered as adequate for S.P. Dun Range to illustrate the efficiency of double ratio estimate of the ratio. Using formulæ (1) and (2)

$$\hat{V} (R'_{yn}) = \cdot 0004951$$

$$\hat{V} (R_{yn}) = \cdot 0010026$$

or % efficiency of double ratio estimate of the ratio is 203.

3. Double Sampling to Estimate the Mean Number of Trees of Lower Diameter Class

A large sample n' of compartments is taken at random and number of trees of higher diameter only are counted, and a sub-sample of size n is taken at random from n' for trees of lower diameter.

Let, y_{1i}, y_{2i} and x_{1i}, x_{2i} denote number of trees in the i-th compartment, of higher and next lower diameter of the present and previous enumerations respectively. If there was no previous complete enumeration, estimate of $\bar{Y}_2$ is given by

$$\hat{\bar{Y}}_2 = \frac{\bar{y}_2}{\bar{y}_1} \cdot \bar{y}_1'$$

where $\bar{y}_2$ and $\bar{y}_1$ are sub-sample means for trees of lower and next higher diameter and $\bar{y}_1'$ the sample mean for trees of higher diameter.

$$V(\hat{\bar{Y}}_2) \approx \left(\frac{1}{n} - \frac{1}{n'}\right) \frac{1}{N-1} \sum_1^N (y_2 - R'_y y_1)^2 + \left(\frac{1}{n'} - \frac{1}{N}\right) \frac{1}{N-1} \sum_1^N (y_2 - \bar{Y}_2)^2$$

where

$$R'_y = \frac{\bar{Y}_2}{\bar{Y}_1}.$$

A consistent estimate of $V(\hat{\bar{Y}}_2)$ is

$$\hat{V}(\hat{\bar{Y}}_2) \approx \left(\frac{1}{n} - \frac{1}{n'}\right) \frac{1}{n-1} \sum_1^n (y_2 - R'_{yn} y_1)^2 + \left(\frac{1}{n'} - \frac{1}{N}\right) \frac{1}{n-1} \sum_1^n (y_2 - \bar{y}_2)^2 \tag{4}$$

where

$$R'_{yn} = \frac{\bar{y}_2}{\bar{y}_1}.$$

If previous complete enumeration data are available, an estimate of $\bar{Y}_2$ utilising the supplementary information of the trees of higher diameter of the present enumeration, is given by

$$\hat{\bar{Y}}_2' = \frac{\bar{y}_2/\bar{x}_2}{\bar{y}_1/\bar{x}_1} \cdot \frac{\bar{y}_1'}{\bar{x}_1'} \cdot \bar{X}_2 = \frac{R_{2n}}{R_{1n}} \cdot \frac{\bar{y}_1'}{\bar{x}_1'} \cdot \bar{X}_2 = \mathrm{R}'_n \cdot \frac{\bar{y}_1'}{\bar{x}_1'} \cdot \bar{X}_2$$

where $\bar{x}_2$ and $\bar{x}_1$ are sub-sample means for trees of lower and next higher diameter, and $\bar{x}_1'$, the sample mean for trees of higher diameter of the previous complete enumeration, and

$$R_{2n} = \frac{\bar{y}_2}{\bar{x}_2}, \quad R_{1n} = \frac{\bar{y}_1}{\bar{x}_1}, \quad R'_n = \frac{R_{2n}}{R_{1n}}.$$

Let,

$$R' = \frac{R_2}{R_1},$$

where

$$R_2 = \frac{\bar{Y}_2}{\bar{X}_2}$$

and

$$R_1 = \frac{\bar{Y}_1}{\bar{X}_1}.$$

Now,

$$\hat{\bar{Y}}_2' - \bar{Y}_2 = \bar{X}_2 \left(\frac{\bar{y}_2/\bar{x}_2}{\bar{y}_1/\bar{x}_1}\right)\left(\frac{\bar{y}_1'}{\bar{x}_1'} - \frac{\bar{Y}_1}{\bar{X}_1}\right) + \bar{X}_2 \frac{(\bar{Y}_1/\bar{X}_1)}{(\bar{y}_1/\bar{x}_1)}$$

$$\times \left(\frac{\bar{y}_2}{\bar{x}_2} - R' \frac{\bar{y}_1}{\bar{x}_1}\right).$$

To get an approximate variance of $\hat{\bar{Y}}_2'$, we follow Cohran (1953) and put,

$$\frac{\bar{Y}_1/\bar{X}_1}{\bar{y}_1/\bar{x}_1} \approx 1$$

and

$$\frac{\bar{y}_2/\bar{x}_2}{\bar{y}_1/\bar{x}_1} \approx R'$$

$$\therefore \quad \hat{\bar{Y}}_2' - \bar{Y}_2 \approx \bar{X}_2 \cdot R' \left(\frac{\bar{y}_1'}{\bar{x}_1'} - \frac{\bar{Y}_1}{\bar{X}_1}\right) + \bar{X}_2 \left(\frac{\bar{y}_2}{\bar{x}_2} - R' \frac{\bar{y}_1}{\bar{x}_1}\right)$$

$$\approx \bar{X}_2 \left(\frac{\bar{y}_2}{\bar{x}_2} - \frac{\bar{Y}_2}{\bar{X}_2}\right) - R' \bar{X}_2 \left(\frac{\bar{y}_1}{\bar{x}_1} - \frac{\bar{y}_1'}{\bar{x}_1'}\right)$$

$$\therefore \quad V(\hat{\bar{Y}}_2') \approx \bar{X}_2^2 \left[V\left(\frac{\bar{y}_2}{\bar{x}_2}\right) - 2 R' \text{ cov.} \left(\frac{\bar{y}_2}{\bar{x}_2} \cdot \frac{\bar{y}_1}{\bar{x}_1}\right)\right.$$

$$+ R'^2 V\left(\frac{\bar{y}_1}{\bar{x}_1}\right) + 2 R' \text{ cov.} \left(\frac{\bar{y}_2'}{\bar{x}_2'}, \frac{\bar{y}_1'}{\bar{x}_1'}\right)$$

$$\left. - R'^2 V\left(\frac{\bar{y}_1'}{\bar{x}_1'}\right)\right].$$

A consistent estimate of $V(\hat{\bar{Y}}_2')$ is given by

$$\hat{V}(\hat{\bar{Y}}_2') \approx \left(\sum_1^n x_2\right)^2 \left[\left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{n-1}\left\{\frac{\sum_1^n (y_2 - R_{2n}x_2)^2}{\left(\sum_1^n x_2\right)^2}\right.\right.$$

$$- 2R'_n \frac{\sum_1^n (y_2 - R_{2n}x_2)(y_1 - R_{1n}x_1)}{\left(\sum_1^n x_1\right)\left(\sum_1^n x_2\right)}$$

$$\left. + R_n'^2 \frac{\sum_1^n (y_1 - R_{1n}x_1)^2}{\left(\sum_1^n x_1\right)^2}\right\} + \left(\frac{1}{n'} - \frac{1}{N}\right)\frac{1}{n-1}$$

$$\times \left\{2R'_n \frac{\sum_1^n (y_2 - R_{2n}x_2)(y_1 - R_{1n}x_1)}{\left(\sum_1^n x_1\right)\left(\sum_1^n x_2\right)}\right.$$

$$\left.\left. - R_n'^2 \frac{\sum_1^n (y_1 - R_{1n}x_1)^2}{\left(\sum_1^n x_1\right)^2}\right\}\right]. \tag{5}$$

An estimate of $\bar{Y}_2$ utilising the supplementary information of the previous complete enumeration only, but not of the number of trees of higher diameter of the present enumeration, is given by

$$\hat{\bar{Y}}_2'' = \frac{\bar{y}_2}{\bar{x}_2} \cdot \bar{X}_2 = R_{2n} \cdot \bar{x}_2$$

and

$$\hat{V}(\hat{\bar{Y}}_2'') \approx \left(\frac{1}{n} - \frac{1}{N}\right)\frac{1}{n-1}\sum_1^n (y_2 - R_{2n}x_2)^2. \tag{6}$$

Therefore, $\hat{\bar{Y}}_2'$ is more precise than $\hat{\bar{Y}}_2''$ if the ratio of the ratios y_2/x_2 and y_1/x_1 is less subject to variation than either ratio separately.

Estimation of mean number of trees per acre of lower diameter class:

Let the variate 'z' denote the area of a compartment or topographical section. If there was no previous complete enumeration, an estimate of mean number of trees per acre of lower diameter class is given by

$$R_{zn} = \frac{\bar{y}_2}{\bar{y}_1} \cdot \frac{\bar{y}_1{}'}{\bar{z}'} = \frac{\bar{y}_2/\bar{z}}{\bar{y}_1/\bar{z}} \cdot \frac{\bar{y}_1{}'}{\bar{z}'} = \frac{R_{2zn}}{R_{1zn}} \cdot \frac{\bar{y}_1{}'}{\bar{z}_1}$$

where $\bar{z}$ and $\bar{z}'$ are sub-sample and sample means of the areas of the units respectively, and

$$R_{2zn} = \frac{\bar{y}_2}{\bar{z}} \quad \text{and} \quad R_{1zn} = \frac{\bar{y}_1}{\bar{z}}.$$

It can be shown by following the technique above, that a consistent estimate of the variance of R_{zn} is given by,

$$\hat{V}(R_{zn}) \approx \frac{1}{\bar{z}^2}\left[\left(\frac{1}{n} - \frac{1}{n'}\right)\frac{1}{n-1}\sum_1^n (y_2 - R'_{yn}y_1)^2 + \left(\frac{1}{n'} - \frac{1}{N}\right)\sum_1^n (y_2 - \bar{y}_2)^2 - \left(\frac{1}{n'} - \frac{1}{N}\right)\frac{1}{n-1} \times \{2R_{2zn}\sum_1^n y_2 z - R^2_{2zn}\sum_1^n z^2 - n\bar{y}_2{}^2\}\right]. \quad (7)$$

If the supplementary information of 'z' is not used, estimate is given by $\hat{\bar{Y}}_2/\bar{Z}$ where $\bar{Z}$ is population mean area and its estimate of the variance is $1/\bar{z}^2\ \hat{V}(\hat{\bar{Y}}_2)$. Therefore, if the ratio y_2/z is less subject to variation than either variate separately, R_{zn} is more precise than $\hat{\bar{Y}}_2/\bar{Z}$. When previous complete enumeration data are available, area factor does not enter into the estimate $\hat{\bar{Y}}_2{}'$ since the estimate can be written as

$$\left[\frac{\bar{y}_2/\bar{z}}{\bar{x}_2/\bar{z}} \bigg/ \frac{\bar{y}_1/\bar{z}}{\bar{x}_1/\bar{z}}\right]\left(\frac{\bar{y}_1{}'/\bar{z}'}{\bar{x}_1{}'/\bar{z}'}\right) \cdot \frac{\bar{X}_2}{\bar{Z}} = \frac{\hat{\bar{Y}}_2{}'}{\bar{Z}}.$$

4. Efficiency of Stratification for the Double Ratio Estimate of the Ratio

When partial enumeration is done using a particular type of stratification, it is of importance to find how much gain of efficiency is obtained due to stratification. Usually, each felling series is divided into several strata, the existing forest blocks being taken as strata, and independent random samples are drawn from each stratum, so that a combined double ratio estimate is to be used to estimate the ratio of class I to II trees. If the values of the ratio of ratios for sampling units within the strata are not uniform, a combined double ratio estimate of the ratio may become less precise than that obtained from an unstratified random sample. Rao (1958) has given a systematic procedure for estimating the efficiency of stratification for the ratio of class I to II trees. Stratification according to contiguous compartments may sometimes result in heterogeneous strata particularly in hilly terrain.

Here formulæ for estimating the efficiency of stratification of the double ratio estimate of the ratio are given. It is to be noted that stratification may be more precise for each of the ratios separately, but may become less precise for the ratio of the ratios, if proper care is not taken beforehand to ensure uniformity for the ratio of the ratios within each stratum.

Let there be k strata in the felling series and $\bar{y}_{1n_t}$, $\bar{y}_{2n_t}$ and $\bar{x}_{1n_t}$, $\bar{x}_{2n_t}$ be sample means of trees of class I and II of the present and previous enumerations respectively for the t-th stratum and n_t, the number of compartments drawn at random from the N_t compartments of the t-th stratum.

A combined double ratio estimate of R_y is given by

$$R'_{yw} = \frac{\sum_1^k p_t\bar{y}_{1n_t} \Big/ \sum_1^k p_t\bar{y}_{2n_t}}{\sum_1^k p_t\bar{x}_{1n_t} \Big/ \sum_1^k p_t\bar{x}_{2n_t}} \cdot \frac{X_1}{X_2}$$

$$= \frac{\bar{y}_{1w}/\bar{y}_{2w}}{\bar{x}_{1w}/\bar{x}_{2w}} \cdot \frac{X_1}{X_2} = \frac{R_{yw}}{R_{xw}} \cdot \frac{X_1}{X_2} = R_w \cdot R_x$$

where

$$p_t = \frac{N_t}{N},$$

whose estimate of variance is

$$\hat{V}(R'_{yw})_S \approx \hat{V}(R_{yw})_S - 2R_w\, \widehat{\text{cov.}}\,(R_{yw}R_{xw})_S$$
$$+ R_w^2\, \hat{V}(R_{xw})_S \tag{8}$$

where 'S' stands for 'stratified'.

$$\hat{V}(R_{yw})_S \approx \frac{1}{\bar{y}^2_{2w}}\left\{\left(\sum_1^k \frac{p_t^2 s_{ty_1}^2}{n_t} - \sum_1^k \frac{p_t^2 s_{ty_1}^2}{N_t}\right) - 2R_{yw}\left(\sum_1^k \frac{p_t^2 s_{ty_1y_2}}{n_t} - \sum_1^k \frac{p_t^2 s_{ty_1y_2}}{N_t}\right) + R_{yw}^2\left(\sum_1^k \frac{p_t^2 s_{ty_2}^2}{n_t} - \sum_1^k \frac{p_t^2 s_{ty_2}^2}{N_t}\right)\right\} \quad (9)$$

a similar expression for $\hat{V}(R_{xw})_S$

and

$$\widehat{\text{cov.}}\ (R_{yw}R_{xw})_S \approx \frac{1}{\bar{y}_{2w}\bar{x}_{2w}}\left\{\left(\sum_1^k \frac{p_t^2 s_{ty_1x_1}}{n_t} - \sum_1^k \frac{p_t^2 s_{ty_1x_1}}{N_t}\right) - R_{yw}\left(\sum_1^k \frac{p_t^2 s_{tx_1y_2}}{n_t} - \sum_1^k \frac{p_t^2 s_{tx_1y_2}}{N_t}\right) - R_{xw}\left(\sum_1^k \frac{p_t^2 s_{ty_1x_2}}{n_t} - \sum_1^k \frac{p_t^2 s_{ty_1x_2}}{N_t}\right) + R_{yw}R_{xw}\left(\sum_1^k \frac{p_t^2 s_{ty_2x_2}}{n_t} - \sum_1^k \frac{p_t^2 s_{ty_2x_2}}{N_t}\right)\right\} \quad \ldots\ldots (10)$$

where $s_{ty_1}^2$, $s_{ty_2}^2$, $s_{ty_1y_2}$ and $s_{ty_ix_i}$ $(i = 1, 2)$ are mean squares and mean products for t-th stratum.

Double ratio estimate of R_y from an unstratified random sample of size n is

$$R'_{yn} = \frac{\bar{y}_1/\bar{y}_2}{\bar{x}_1/\bar{x}_2} \cdot \frac{\bar{X}_1}{\bar{X}_2} = \frac{R_{yn}}{R_{xn}} \cdot R_x = R_n \cdot R_x$$

which is not known.

Sukhatme (1954) has shown that

$$\hat{V}(\bar{y})_{U.S} \approx \left(\frac{1}{n} - \frac{1}{N}\right)\left\{\sum_1^k p_t s_{ty}^2 + \sum_1^k p_t \bar{y}_{nt}^2 - \left(\sum_1^k p_t \bar{y}_{nt}\right)^2 - \left(\sum_1^k \frac{p_t s_{ty}^2}{n_t} - \sum_1^k \frac{p_t^2 s_{ty}^2}{n_t}\right)\right\} \tag{11}$$

where ' $U.S$' stands for ' unstratified '.

It can be shown that

$$\widehat{\text{cov}}.\ (\bar{y}\bar{x})_{U.S} \approx \left(\frac{1}{n} - \frac{1}{N}\right)\left\{\sum_1^k p_t s_{tyx} + \sum_1^k p_t \bar{y}_{nt}\bar{x}_{nt} - \left(\sum_1^k p_t \bar{y}_{nt}\right)\left(\sum_1^k p_t \bar{x}_{nt}\right) - \left(\sum_1^k \frac{p_t s_{tyx}}{n_t} - \sum_1^k \frac{p_t^2 s_{tyx}}{n_t}\right)\right\} \tag{12}$$

$$\therefore\quad \hat{V}(R'_{yn})_{U.S} \approx \hat{V}(R_{yn})_{U.S} - 2R_w\, \widehat{\text{cov}}.\ (R_{yn}R_{xn})_{U.S} + R_w^2\, \hat{V}(R_{xn})_{U.S} \tag{13}$$

where

$$\hat{V}(R_{yn})_{U.S} \approx \frac{1}{\bar{y}_{2w}^2}\{\hat{V}(\bar{y}_1)_{U.S} - 2R_{yw}\, \widehat{\text{cov}}.\ (\bar{y}_1\bar{y}_2)_{U.S} + R_{yw}^2\, \hat{V}(\bar{y}_2)_{U.S}) \tag{14}$$

a similar expression for $\hat{V}(R_{xn})_{U.S}$ and

$$\widehat{\text{cov}}.\ (R_{yn}R_{xn})_{U.S} \approx \frac{1}{\bar{y}_{2w}\bar{x}_{2w}}(\widehat{\text{cov}}.\ (\bar{y}_1\bar{x}_1)_{U.S} - R_{yw}\, \widehat{\text{cov}}.\ (\bar{y}_2\bar{x}_1)_{U.S} - R_{xw}\, \widehat{\text{cov}}.\ (\bar{y}_1\bar{x}_2)_{U.S} + R_{yw}R_{xw}\, \widehat{\text{cov}}.\ (\bar{y}_2\bar{x}_2)_{U.S}\} \tag{15}$$

and variances and covariances can be substituted from equations (11) and (12) in (13).

To compute $\hat{V}(R_{yn})_{U.S}$ and $\hat{V}(R_{yw})_S$, we follow the systematic procedure given in Rao (1958), and form variance tables of 'y_1' and 'y_2' to get $\hat{V}(\bar{y}_1)_{U.S}$, $\hat{V}(\bar{y}_{1w})_S$ and $\hat{V}(\bar{y}_2)_{U.S}$ and $\hat{V}(\bar{y}_{2w})_S$, and covariance table of 'y_1' and 'y_2' to get $\widehat{\text{cov}}.\ (\bar{y}_1\, \bar{y}_2)_{U.S}$ and $\widehat{\text{cov}}.\ (\bar{y}_{1w}\bar{y}_{2w})_S$ and substitute these values in equations (14) and (9). Similarly for $\hat{V}(R_{xn})_{U.S}$ and $\hat{V}(R_{xw})_S$. To compute $\widehat{\text{cov}}.\ (R_{yn}R_{xn})_{U.S}$ and $\widehat{\text{cov}}.\ (R_{yw}R_{xw})_S$, we form covariance tables of 'y_1' and 'x_1', 'y_2' and 'x_2', 'y_2' and 'x_1' and 'y_1' and 'x_2' to get $\widehat{\text{cov}}.\ (\bar{y}_1\,\bar{x}_1)_{U.S}$ and $\widehat{\text{cov}}.\ (y_{1w}\bar{x}_{1w})_S$, $\widehat{\text{cov}}.\ (\bar{y}_2\,\bar{x}_2)_{U\,S}$ and $\widehat{\text{cov}}.\ (\bar{y}_{2w}\bar{x}_{2w})_S$, $\widehat{\text{cov}}.\ (\bar{y}_2\bar{x}_1)_{U\,S}$ and $\widehat{\text{cov}}.\ (\bar{y}_{2w}\bar{x}_{1w})_S$ and $\widehat{\text{cov}}.\ (\bar{y}_1\bar{x}_2)_{U.S}$ and $\widehat{\text{cov}}.\ (\bar{y}_{1w}\bar{x}_{2w})_S$ respectively and substitute these values in equations (15) and (10).

SUMMARY

The use of double ratio estimate to estimate the ratio of class I to II trees when previous complete enumeration data are available is illustrated. Double sampling is used for trees of the lower diameter class. The estimate and the variance for mean number of trees of lower diameter class utilising the supplementary information of the number of trees of higher diameter class, or the previous complete enumeration data or both are given. Efficiency of stratification for the double ratio estimate of the ratio of class I to II trees is also considered.

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